## The importance of weak boson emission at LHC

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Abstract: We point out that gauge bosons emissions should be carefully estimated when considering LHC observables, since real $W s$ and $Z s$ contributions can dramatically change cross sections with respect to tree level values. Here we consider observables that are fully inclusive respect to soft gauge boson emission and where a certain number of nonabelian isospin charges in initial and/or final states are detected. We set up a general formalism to evaluate leading, all order resummed electroweak corrections and we consider the phenomenologically relevant case of third family quark production at the LHC. In the case of $b \bar{t}$ production we find that, due to the interplay between strong and weak interactions, the production cross section can become an order of magnitude bigger than the tree level value.

Keywords: Standard Model, Hadronic Colliders.

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## 1. Introduction

It is by now well established that one loop electroweak corrections are not sufficient to keep under control Standard Model predictions at the TeV scale. The reason for this is the sharp growth with energy of these kind of corrections, that reach the $10 \%$ level at 1 TeV . More in detail, this growth is related to the infrared structure of the theory, so that one loop contributions are proportional to $\log ^{2} \frac{s}{M_{W, Z}^{2}}$, the gauge bosons masses $M_{W, Z}$ acting as infrared regulators [1]. In order to cope with the expected precision of hadronic (LHC) and leptonic (ILC) colliders, fixed-order one and two loop corrections 2 and resummation of leading effects [3] have been considered by various groups in the last few years. Broadly speaking, two kinds of observables have been considered. In first place the exclusive observables where only virtual electroweak effects need to be considered [2, 3. In second place, observables including $W, Z, \gamma$ emissions have been considered [ 6 . A prototype of these kind of observables is $e^{+} e^{-} \rightarrow$ hadrons: the two final jets are detected, while any other object in the final state is summed over, including the final decay products of $W s$ and $Z s$ [東]. In this case, the collider provides two initial nonabelian charges, and due to this fact the outcome is surprising: even though fully inclusive, the cross section is sensitive to the infrared cutoff $M_{W}$ and affected by big $\log ^{2} \frac{s}{M_{W}^{2}}$ terms. This effect, baptized "BlochNordsieck violation", has been shown to occur only in broken gauge theories, including the abelian case [5]. Recently, electroweak evolution equations, which are the analogous of QCD DGLAP equations, have been derived [6].

The aim of this work is to considered another class of observables, which we might call "partially inclusive". That is, gauge boson radiation in is still summed over, however we retain the possibility to observe nonabelian charges in both the initial and/or final states. The cross sections we consider are, generally speaking, identified by 4 hard partons, 2 in the initial and 2 in the final state. However in order to compute the leading logs related to virtual and real $W, Z, \gamma$ emission we can reduce to 2 or 3 relevant legs. Suppose for instance


Figure 1: Diagrammatic Isospin description of the overlap matrix with three isospin charged legs and one inclusive final state.
that the flavor of leg 4 is undetected (see fig.1) This could be the case if we consider leg 4 to be a final jet and we do not isolate the jet's flavor. Then, by unitarity, since we are summing over real and virtual corrections [4, we need only to consider the electroweak corrections related to legs $1,2,3$. This is showed in fig. 1 where the overlap matrix with the remaining three legs is depicted.

## 2. General formalism

Our starting point, as in previous works, is the $\mathrm{SU}(2)$ isospin structure of the overlap matrix, defined in terms of the S-matrix as follows $\mathbb{\sharp}\left(\alpha_{i}, \beta_{i}\right.$ are the isospin indices):

$$
\begin{equation*}
\left\langle\beta_{1} \beta_{2} \ldots \beta_{n}\right| S^{\dagger} S\left|\alpha_{1} \alpha_{2} \ldots \alpha_{n}\right\rangle=\mathcal{O}_{\beta_{1} \alpha_{1}, \beta_{2} \alpha_{2}, \ldots \beta_{n} \alpha_{n}} \tag{2.1}
\end{equation*}
$$

and the observable cross sections are related to the above definition by:

$$
\begin{equation*}
d \sigma_{\alpha_{1} \alpha_{2} \ldots \alpha_{n}}=\mathcal{O}_{\alpha_{1} \alpha_{1}, \alpha_{2} \alpha_{2}, \ldots \alpha_{n} \alpha_{n}} \tag{2.2}
\end{equation*}
$$

Notice that we use a "generalized S matrix" formalism, such that all the states over which we are inclusive appear on the left of S and all the detected nonabelian charges $1,2 \ldots n$ appear on the right. So for instance a detected final (outgoing) antiquark is seen as an initial (ingoing) quark state. Namely, for the case $n=2$ this means that we describe cross sections with two electroweak charges in the initial states or systems with one charged particle in the initial and one in the final state, or the case of two final charges.

The $\operatorname{SU}(2)$ generators $t^{a}, a=1,2,3 \quad i=1,2, \ldots n$ act on the overlap matrix as follows

$$
\begin{equation*}
\left(t_{i}^{a} \mathcal{O}\right)_{\beta_{1} \alpha_{1}, \ldots \beta_{n} \alpha_{n}}=\sum_{\delta_{i}} t_{\alpha_{i} \delta_{i}}^{a} \mathcal{O}_{\beta_{1} \alpha_{1}, \ldots \beta_{i} \delta_{i}, \ldots \beta_{n} \alpha_{n}} \quad\left(t_{i}^{\prime a} \mathcal{O}\right)_{\beta_{1} \alpha_{1}, \ldots \beta_{n} \alpha_{n}}=\sum_{\gamma_{i}} t_{\beta_{i} \gamma_{i}}^{a} \mathcal{O}_{\beta_{1} \alpha_{1}, \ldots \gamma_{i} \alpha_{i}, \ldots \beta_{n} \alpha_{n}} \tag{2.3}
\end{equation*}
$$

where the generators $t^{a}$ depend on the representation of the considered $i-t h$ particle.
It is convenient to define the isospin generator referred to a single leg $i$ as $T_{i} \equiv t_{i}+$ $t_{i}^{\prime}$ [4]. Since we consider energy scales of the order of 1 TeV and beyond, we take all particles to be massless. In other words we work in the high energy limit in which the
$S U(2) \otimes U(1)$ symmetry is recovered; then the overlap matrix is invariant under a symmetry transformation:

$$
\begin{equation*}
T_{t o t}^{a} \equiv \sum_{i} T_{i}^{a} \quad \exp \left[\alpha^{a} T_{t o t}^{a}\right] \mathcal{O}=\exp \left[\boldsymbol{\alpha} \cdot \boldsymbol{T}_{t o t}\right] \mathcal{O}=\mathcal{O} \quad \Rightarrow \quad \boldsymbol{T}_{t o t} \mathcal{O}=0 \tag{2.4}
\end{equation*}
$$

This property allows to write the overlap matrix as a sum of projectors with definite isospin properties and gives various relations between the apriori independent cross sections (see next sections).

The dressing of the hard overlap matrix $\mathcal{O}^{H}$ to obtain the evolved one $\mathcal{O}$ through exchange of virtual and real soft $W$ quanta is described by the external line insertion of the eikonal current:

$$
\begin{equation*}
\boldsymbol{J}^{\mu}(k)=g_{w} \sum_{i=1}^{n} \boldsymbol{T}_{i} \frac{p_{i}^{\mu}}{p_{i} \cdot k} \tag{2.5}
\end{equation*}
$$

$k$ being the momentum of the emitted soft gauge boson, $p_{i}$ the i-th leg momentum and $g_{w}$ the $\mathrm{SU}(2)$ gauge coupling. Notice that the part of the current proportional to $g^{\prime}$ is absent altogether because of the cancellation of the abelian components for inclusive observables (4).

By squaring the eikonal current one obtains, in the limit where all invariants are of the same order $2 p_{i} \cdot p_{j} \approx s$, the insertion operator:

$$
\begin{equation*}
I(k)=g_{w}^{2} \frac{p_{1} p_{2}}{\left(k p_{1}\right)\left(k p_{2}\right)} \sum_{i<j}^{n} \boldsymbol{T}_{i} \cdot \boldsymbol{T}_{j} \tag{2.6}
\end{equation*}
$$

The resummed expression for the overlap matrix is finally given by the following expression involving the insertion operator:

$$
\begin{equation*}
\mathcal{O}(s)=\exp \left[L_{W} \sum_{i<j}^{n} \boldsymbol{T}_{i} \cdot \boldsymbol{T}_{j}\right] \mathcal{O}^{H} \tag{2.7}
\end{equation*}
$$

where we have defined the eikonal radiation factor for $W$ exchange:

$$
\begin{equation*}
L_{W}(s)=\frac{g_{w}^{2}}{2} \int_{M}^{E} \frac{d^{3} \boldsymbol{k}}{2 \omega_{k}(2 \pi)^{3}} \frac{2 p_{1} p_{2}}{\left(k p_{1}\right)\left(k p_{2}\right)}=\frac{\alpha_{w}}{4 \pi} \log ^{2} \frac{s}{M^{2}}, \quad \alpha_{w}=\frac{g_{w}^{2}}{4 \pi} \tag{2.8}
\end{equation*}
$$

It is useful to rewrite (2.7) by using

$$
\begin{equation*}
\sum_{i<j}^{n} \boldsymbol{T}_{i} \cdot \boldsymbol{T}_{j}=\frac{1}{2} \sum_{i=1}^{n} \boldsymbol{T}_{i} \cdot\left(\boldsymbol{T}_{t o t}-\boldsymbol{T}_{i}\right)=-\frac{1}{2} \sum_{i} \boldsymbol{T}_{i}^{2} \tag{2.9}
\end{equation*}
$$

where we used $\boldsymbol{T}_{t o t} \mathcal{O}=0$, so that

$$
\begin{equation*}
\mathcal{O}(s)=\exp \left[-\frac{1}{2} L_{W} \sum_{i=1}^{n} \boldsymbol{T}_{i}^{2}\right] \mathcal{O}^{H} \tag{2.10}
\end{equation*}
$$

Eqn. (2.10) shows the single particle property of inclusive emission, i.e. the fact that corrections are calculated by considering an exponential factor for each leg in a definite
total isospin state. In the following we systematically adopt the procedure of writing the overlap matrix as a sum of projectors on total isospin eigenstates and then applying (2.10) to obtain the "EW BN" corrected overlap matrix. The hard overlap matrix is therefore first decomposed as follows:

$$
\begin{equation*}
\mathcal{O}^{H}=\sum_{t_{1}, t_{2} \ldots t_{n}} O_{t_{1} t_{2} \ldots t_{n}}^{H} \mathcal{P}_{t_{1} t_{2} \ldots t_{n}} \tag{2.11}
\end{equation*}
$$

where $\mathcal{O}, \mathcal{P}_{t_{1} t_{2} \ldots t_{n}}$ are operators acting on the $n$ external legs indices, and $O_{t_{1} t_{2} \ldots t_{n}}$ are the coefficients of the expansion. The projectors satisfy, by definition:

$$
\begin{equation*}
\boldsymbol{T}_{j} \mathcal{P}_{t_{1} t_{2} \ldots t_{n}}=t_{j} \mathcal{P}_{t_{1} t_{2} \ldots t_{n}}, j=1 \ldots n \quad \boldsymbol{T}_{t o t} \mathcal{P}_{t_{1} t_{2} \ldots t_{n}}=0 \tag{2.12}
\end{equation*}
$$

Then we apply (2.10) in order to obtain the all order resummed values. Due to property (2.12) this is particularly simple, since it amounts to the substitution:

$$
\begin{equation*}
O_{t_{1} t_{2} \ldots t_{n}}^{H} \rightarrow O_{t_{1} t_{2} \ldots t_{n}}(s)=\exp \left[-\frac{1}{2} L_{W}(s) \sum_{i=1}^{n} t_{i}\left(t_{i}+1\right)\right] O_{t_{1} t_{2} \ldots t_{n}}^{H} \tag{2.13}
\end{equation*}
$$

In next section we give the explicit form of the projection operators for the cases of two and three external legs.

To end with, we want to compare the above describe "BN EW" corrections with "Sudakov EW" corrections, i.e. EW corrections given only by the virtual contributions without weak bosons emissions. The latter depend on how the observable is defined, namely on which cutoff is decided on real photon emission: a certain degree of inclusiveness on photons is mandatory in order to render the observable infrared finite. For definiteness and in order to compare with the BN corrections, we choose for the photon a cutoff of the order of the weak scale; the result is an effective $\mathrm{SU}(2) \otimes \mathrm{U}(1)$ theory with all gauge bosons at a common mass $M_{W} \approx M_{Z}$ [3]. In this limit Sudakov corrections are in fact rather simple: the resummed cross section is obtained from the hard one by multiplying each external leg by an exponential factor:

$$
\begin{equation*}
\sigma^{S u d}(s)=\exp \left[-L_{W}(s) \sum_{i}\left(t_{i}\left(t_{i}+1\right)+y_{i}^{2} \tan ^{2} \theta_{W}\right)\right] \sigma_{H} \tag{2.14}
\end{equation*}
$$

where $\theta_{W}$ is the Weinberg angle, $t_{i}$ is the i-th leg isospin and $y_{i}$ its hypercharge [3]. Despite the similarities between $(\boxed{2.13})$ and $(\boxed{2.14})$, the inclusive $(\mathrm{BN})$ and exclusive (Sudakov) case are of course very different and give rise to significantly different patterns of radiative corrections. Namely:

- in (2.14) $t_{i}$ is the external leg isospin (e.g., $\frac{1}{2}$ for a fermion) while in (2.13) $t_{i}$ is obtained by composing two single-leg isospins (see fig. 1)
- no correction proportional to $y^{2}$ is present in the " BN " inclusive case, since contributions proportional to the $\mathrm{U}(1)$ coupling $g_{y}$ cancel out [4].
- There is a factor 2 of difference in the argument of the exponential (compare (2.14), (2.13)).
- while Sudakov corrections always depress the tree level cross section, BN ones can be negative or positive (see section 4).


## 3. The case of two and three external legs

In this section we give the explicit forms of the projectors in the case of two and three external legs. This allows to calculate the EW BN corrections by simply inserting the appropriate values of the hard cross sections, as we explain in next section. Of course not all of the possible values of $t_{1}, t_{2} \ldots t_{n}$ appearing in (2.11) are allowed. In fact the total isospin must be 0 due to isospin invariance, so for instance in the case of two legs there is no $\mathcal{P}_{10}$ term, since no isospin invariant can be constructed from an isospin 0 and an isospin 1 objects. In the following we present tables with the allowed values for $t_{1} \ldots t_{n}$. The coefficients $O_{t_{1} t_{2} \ldots t_{n}}$ are also called "form factors" since they are $s$-dependent and receive the exponential factor (2.13). Here we limit ourselves to fermions, antifermions and transverse gauge bosons in the external legs. Therefore in the following by "boson" we always mean "transversely polarized (weak) gauge boson".

We now consider the "two external legs" case. Notice that by this we do not mean that only two external particles are present. Rather, we mean a process with an arbitrary number of external particles, but in which only two non abelian weak charges are detected. Therefore, the process $g g \rightarrow q \bar{q}$ belongs to this category since gluons do not carry weak charges; also the process $e^{+} e^{-} \rightarrow$ jets $+X$ falls in this case since this process is fully inclusive in the final state: no weak charge is singled out. The case of two initial external charged legs has been widely discussed from various point of view: in 图 the BN violation was put in evidence and more refined studies were done in [5] including the next to leading corrections. Here we extend the same procedure to processes with two external charged legs irrespective of their position of initial or final states.

The possible values for the isospin of the two external legs labeled by $t_{1}, t_{2}$ are given by (f.f. $=$ form factors):

| fermion | fermion | Number of f.f. | fermion | boson | Number of f.f. | boson | boson | Number of f.f. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $t_{1}=0$ | $t_{2}=0$ |  | $t_{1}=0$ | $t_{2}=0$ |  | $t_{1}=0$ | $t_{2}=0$ |  |
| $t_{1}=1$ | $t_{2}=1$ |  | $t_{1}=1$ | $t_{2}=1$ | 2 | $t_{1}=1$ | $t_{2}=1$ | 3 |
| $t_{1}=2$ | $t_{2}=2$ |  |  |  |  |  |  |  |

The explicit form of the projection operators is given below for the relevant case of fermions and (transverse) gauge bosons:

- two external fermions

$$
\begin{equation*}
\mathcal{O}\left(\alpha_{1}, \beta_{1} ; \alpha_{2}, \beta_{2}\right)=O_{00} \delta_{\alpha_{1} \beta_{1}} \delta_{\alpha_{2} \beta_{2}}+O_{11} t_{\alpha_{1} \beta_{1}}^{a} t_{\alpha_{2} \beta_{2}}^{a} \tag{3.1}
\end{equation*}
$$

- one fermion and one external gauge boson

$$
\begin{equation*}
\mathcal{O}\left(\alpha_{1}, \beta_{1} ; a_{2}, b_{2}\right)=O_{00} \delta_{\alpha_{1} \beta_{1}} \delta_{a_{2} b_{2}}+O_{11} t_{\alpha_{1} \beta_{1}}^{a} T_{a_{2} b_{2}}^{a} \tag{3.2}
\end{equation*}
$$

- two external gauge bosons

$$
\begin{equation*}
\mathcal{O}\left(a_{1}, b_{1} ; a_{2}, b_{2}\right)=O_{00} \delta_{a_{1} b_{1}} \delta_{a_{2} b_{2}}+O_{11} T_{a_{1} b_{1}}^{a} T_{a_{2} b_{2}}^{a}+O_{22} \mathcal{P}_{2}\left(a_{1} b_{1} ; a_{2} b_{2}\right) \tag{3.3}
\end{equation*}
$$

where $t^{a}\left(T^{a}\right)$ are the $\mathrm{SU}(2)$ generators in the fundamental (adjoint) representation and the isospin 2 projector is defined by:

$$
\begin{equation*}
\mathcal{P}_{2}\left(a_{1}, b_{1} ; a_{2}, b_{2}\right)=\frac{1}{4}\left[\left\{T^{c}, T^{d}\right\}_{b_{1} a_{1}}\left\{T^{c}, T^{d}\right\}_{b_{2} a_{2}}-\frac{16}{3} \delta_{b_{1} a_{1}} \delta_{b_{2} a_{2}}\right] \tag{3.4}
\end{equation*}
$$

The case with three external particles charged under $S U(2)$ is more complicated because the product of three isospins generates many invariant with definite total isopin. As shown in the table below in order to describe a system with three external fermion in the fundamental representation five gauge invariant form factors are needed; a system with two fundamental fermions and one boson (in the adjoint representation ) needs six form factors and for a system with one fermion plus two bosons we have to write nine form factors.

| fermion | fermion | fermion | Number of f.f. | fermion | fermion | boson | Number of f.f. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $t_{1}=0$ | $t_{2}=0$ | $t_{3}=0$ |  | $t_{1}=0$ | $t_{2}=0$ | $t_{3}=0$ |  |
| $t_{1}=1$ | $t_{2}=0$ | $t_{3}=1$ | 5 | $t_{1}=1$ | $t_{2}=0$ | $t_{3}=1$ | 6 |
| $t_{1}=0$ | $t_{2}=1$ | $t_{3}=1$ |  | $t_{1}=0$ | $t_{2}=1$ | $t_{3}=1$ |  |
| $t_{1}=1$ | $t_{2}=1$ | $t_{3}=0,1$ |  | $t_{1}=1$ | $t_{2}=1$ | $t_{3}=0,1,2$ |  |


| fermion | boson | boson | Number of f.f. |
| :---: | :---: | :---: | :---: |
| $t_{1}=0$ | $t_{2}=0$ | $t_{3}=0$ |  |
| $t_{1}=1$ | $t_{2}=0$ | $t_{3}=1$ |  |
| $t_{1}=0$ | $t_{2}=1$ | $t_{3}=1$ | 9 |
| $t_{1}=1$ | $t_{2}=1$ | $t_{3}=0,1,2$ |  |
| $t_{1}=0$ | $t_{2}=2$ | $t_{3}=2$ |  |
| $t_{1}=1$ | $t_{2}=2$ | $t_{3}=1,2$ |  |

For the case with three bosons there are 15 form factors. The form factors are gauge invariant combinations of physical cross sections (that correspond to the diagonal elements of the overlap matrix). Reversing the problem, any physical cross section is a combination of form factors. It is interesting to note that the form factors $\mathcal{O}_{i j k \ldots}$ whose sum $i+j+k+\ldots=$ odd number do not contribute to physical cross sections. In practice this means that the degrees of freedom of the overlap matrix projected on the physical space of the cross sections are diminished. In the above examples, with three external particles the form factors $\mathcal{O}_{111}$ and $\mathcal{O}_{122}$ are unphysical. The final result is that, at the level of physical cross sections, the system with three fermions has 4 degrees of freedom: once we know 4 cross sections any other one is fixed as a combination of these ones. The system of two fermions and one boson has 5 degrees of freedom and finally the one fermion and two bosons only 7.

The isospin decomposition of the overlap matrix in the various cases is given below. Fermionic and antifermionic legs are treated on equal grounds, so for instance the case of one antifermionic and two fermionic legs belongs to the " 3 fermionic legs" case.

- 3 fermionic legs

$$
\begin{align*}
\mathcal{O}\left(\alpha_{1}, \beta_{1} ; \alpha_{2}, \beta_{2} ; \alpha_{3}, \beta_{3}\right)= & O_{000} \delta_{\alpha_{1} \beta_{1}} \delta_{\alpha_{2} \beta_{2}} \delta_{\alpha_{3} \beta_{3}}+O_{101} t_{\alpha_{1} \beta_{1}}^{a} \delta_{\alpha_{2} \beta_{2}} t_{\alpha_{3} \beta_{3}}^{a} \\
& +O_{111} f_{a b c} t_{\alpha_{1} \beta_{1}}^{a} t_{\alpha_{2} \beta_{2}}^{b} t_{\alpha_{3} \beta_{3}}^{c} \\
& +O_{011} \delta_{\alpha_{1} \beta_{1}} t_{\alpha_{2} \beta_{2}}^{a} t_{\alpha_{3} \beta_{3}}^{a}+O_{110} t_{\alpha_{1} \beta_{1}}^{a} t_{\alpha_{2} \beta_{2}}^{a} \delta_{\alpha_{3} \beta_{3}} \tag{3.5}
\end{align*}
$$

- 2 fermionic, 1 bosonic

$$
\begin{aligned}
\mathcal{O}\left(\alpha_{1}, \beta_{1} ; \alpha_{2}, \beta_{2} ; a_{3}, b_{3}\right)= & O_{000} \delta_{\alpha_{1} \beta_{1}} \delta_{\alpha_{2} \beta_{2}} \delta_{a_{3} b_{3}}+O_{101} t_{\alpha_{1} \beta_{1}}^{a} \delta_{\alpha_{2} \beta_{2}} T_{a_{3} b_{3}}^{a} \\
& +O_{011} \delta_{\alpha_{1} \beta_{1}} t_{\alpha_{2} \beta_{2}}^{a} T_{a_{3} b_{3}}^{a}
\end{aligned}
$$

$$
\begin{align*}
& +O_{110} t_{\alpha_{1} \beta_{1}}^{a} t_{\alpha_{2} \beta_{2}}^{a} \delta_{a_{3} b_{3}}+O_{112} t_{\alpha 1 \beta 1}^{a} t_{\alpha 2 \beta 2}^{b} \mathcal{P}_{2}\left(a, b ; a_{3}, b_{3}\right) \\
& +O_{111} f_{a b c} t_{\alpha_{1} \beta_{1}} t_{\alpha_{2} \beta_{2}}^{b} T_{a_{3} b_{3}}^{c} \tag{3.6}
\end{align*}
$$

- 1 fermionic, 2 bosonic

$$
\begin{align*}
\mathcal{O} & =O_{000} \delta_{\alpha_{1} \beta_{1}} \delta_{a_{2} b_{2}} \delta_{a_{3} b_{3}}+O_{022} \delta_{\alpha_{1} \beta_{1}} \mathcal{P}_{2}\left(a_{2}, b_{2} ; a_{3}, b_{3}\right)+O_{101} \delta_{a_{2} b_{2}} t_{\alpha_{1} \beta_{1}}^{a} T_{a_{3} b_{3}}^{a} \\
& +O_{110} \delta_{a_{3} b_{3}} t_{\alpha_{1} \beta_{1}}^{a} T_{a_{2} b_{2}}^{a}+O_{111} f_{a b c} t_{\alpha_{1} \beta_{1}}^{a} T_{a_{2} b_{2}}^{b} T_{a_{3} b_{3}}^{c}+O_{112} t_{\alpha_{1} \beta_{1}}^{a} T_{a_{2} b_{2}}^{b} \mathcal{P}_{2}\left(a, b ; a_{3}, b_{3}\right) \\
& +O_{121} t_{\alpha_{1} \beta_{1}}^{a} T_{a_{3} b_{3}}^{b} \mathcal{P}_{2}\left(a, b ; a_{2}, b_{2}\right)+O_{121} t_{\alpha_{1} \beta_{1}}^{a} T_{a_{3} b_{3}}^{b} \mathcal{P}_{2}\left(a, b ; a_{2}, b_{2}\right) \\
& +O_{122} f_{a b c} t_{\alpha_{1} \beta_{1}}^{a} \mathcal{P}_{2}\left(b, d ; a_{2}, b_{2}\right) \mathcal{P}_{2}\left(c, d ; a_{3}, b_{3}\right) \tag{3.7}
\end{align*}
$$

The $\mathrm{SU}(2)$ symmetry encoded into eqns. (3.5-3.7) gives various relations between the apriori independent overlap matrix elements, and therefore between the various cross sections. For instance in the case of (3.5) the overlap has $2^{6}=64$ values in principle; however there are only 4 independent projectors, and therefore only 4 independent values. The most general relation is the following: let us give the index assignments:

$$
\begin{equation*}
1=\nu, 2=e \text { for fermions } \quad 1=W^{+}, 2=W^{-}, 3=W^{3} \text { for gauge bosons } \tag{3.8}
\end{equation*}
$$

then we have $\sigma_{a b c}=\sigma_{a^{\prime} b^{\prime} c^{\prime}}$ where the set $a^{\prime} b^{\prime} c^{\prime}$ is obtained from $a b c$ by the exchange $1 \leftrightarrow 2,3 \leftrightarrow 3$. So $\sigma_{112}=\sigma_{221}, \sigma_{331}=\sigma_{332}$ and so on. It is easy to realize that the described exchange corresponds to a unitary transformation with the matrix $i \sigma_{2}$. The dressing of the overlap matrix, i.e. resumming leading electroweak double logs at all orders, is done by applying eq. (2.10). The above formula will be useful in order to evaluate the BN corrections to the high energy cross section for third family quarks at LHC.

## 4. Third generation quarks production at LHC

In this section we will apply some of the above formulas for LHC cross sections partially inclusive over soft $W$ and $Z$ emission. The idea is to analyze the third family (top and bottom) quark production at LHC for very large momentum transfer. In order to give an idea of the size of the corrections we will also compute the resummed Sudakov corrections at leading order. In such a way we can compare processes without any emission of $W$ and processes with the same hard final states but with the possibility to emit soft $W$ bosons.

As is well known we can write the heavy quark production mechanism at LHC as a convolution of the luminosity functions $L_{i j}$ for the partons $p_{i}$ and $p_{j}$ times the partonic cross sections [7]:

$$
\begin{equation*}
\frac{d \sigma_{P P \rightarrow Q \bar{Q}}}{d \hat{s}}=\frac{1}{s} \sum_{i, j} L_{i j}(\hat{s}) d \sigma_{p_{i} p_{j} \rightarrow Q \bar{Q}}(\hat{s}) \quad L_{i j}(\hat{s})=\int_{\frac{\hat{s}}{s}}^{1} \frac{d x}{x} f_{p_{i}}(x) f_{p_{j}}\left(\frac{\hat{s}}{s x}\right) \tag{4.1}
\end{equation*}
$$

where $f_{p_{i}}(x)$ is the distribution of parton $i$ inside the proton, $\sqrt{\hat{s}}$ is the partonic c.m. energy and $\sqrt{s}=14 \mathrm{TeV}$ the hadronic one. For each channel we will decompose the hard partonic cross sections in isospin defined form factors whose EW BN corrections can be directly computed with eq. (2.13).


Figure 2: Parton distributions for the sea of the proton at $Q=4000 \mathrm{GeV}$ as a function of $x$, the fraction of momentum carried by the parton. Left: $s=\bar{s}, c=\bar{c}, b=\bar{b}$. Right: $\bar{u}, \bar{d}$.

At this point analyzing the luminosity functions of the sea quarks of the proton we can obtain a quite reasonable simplification for the evaluation of the BN corrections to the $q \bar{q}$ cross section of eq. (4.1): the proton sea is approximately an isospin singlet. In other word the amount of anti up quark inside a proton is almost the same of the antiquark down and so on for the other sea families. This statement, from the $S U(2)$ point of view, implies automatically that, with a reasonable approximation, the sea quarks of a proton is a flavour singlet state. To corroborate our statement we show in fig.2) the $x$ dependence of structure functions of the anti-up and anti-down quarks and of the remaining sea quarks at fixed energy. This implies that the partonic process $q \bar{q} \rightarrow Q \bar{Q}$ is, with a good approximation, a three leg process and not a four legs one, since the $\bar{q}$ leg is summed over $\mathrm{SU}(2)$ quantum numbers, and therefore receives no inclusive EW corrections (see also fig. 1).

Let us now turn to the expression for tree level (hard) partonic cross section $q \bar{q} \rightarrow Q \bar{Q}$, where $q(Q)$ is a light (heavy) quark, and we sum over the initial antiquark isospin. Our notation is $\alpha_{1}=1,2$ for up and for down type initial quarks, $\alpha_{2}=1,2$ for $t, b$ final states and $\alpha_{3}=1,2$ for $\bar{t}, \bar{b}$ antifermion final states. The $\mathrm{SU}(3) \otimes \mathrm{SU}(2) \otimes \mathrm{U}(1)$ couplings are given respectively by $g_{s}, g_{w}, g_{y}$. In the massless limit chirality is conserved, so we can label the cross sections with the chirality of the initial and final fermions: the chiralities of corresponding antifermions are unambiguously fixed. The overlap matrix receives various
contributions from $s$-channel exchange of electroweak gauge bosons and gluons*:

$$
\begin{align*}
\frac{d \sigma_{L L}^{H}\left(\alpha_{1}, \alpha_{2}, \alpha_{3}\right)}{d \cos \theta}= & \frac{\hat{u}^{2}}{8 \pi \hat{s}^{3}}\left[\left(g_{y}^{4} y_{L \alpha_{1}}^{2} y_{L \alpha_{2}}^{2}+2 g_{y}^{2} g_{w}^{2} y_{L \alpha_{1}} y_{L \alpha_{2}} t_{\alpha_{1} \alpha_{1}}^{3} t_{\alpha_{2} \alpha_{3}}^{3}\right) \delta_{\alpha_{2} \alpha_{3}}\right.  \tag{4.2}\\
& \left.+g_{w}^{4} t_{\alpha_{3} \alpha_{2}}^{c}\left(t^{c} t^{d}\right)_{\alpha_{1} \alpha_{1}} t_{\alpha_{2} \alpha_{3}}^{d}+\frac{2}{9} g_{s}^{4} \delta_{\alpha_{2} \alpha_{3}}\right] \\
\frac{d \sigma_{L R}^{H}\left(\alpha_{1}, \alpha_{2}, \alpha_{3}\right)}{d \cos \theta}= & \frac{\hat{t}^{2}}{8 \pi \hat{s}^{3}}\left[g_{y}^{4} y_{L \alpha_{1}}^{2} y_{R \alpha_{2}}^{2}+\frac{2}{9} g_{s}^{4}\right] \delta_{\alpha_{2} \alpha_{3}}  \tag{4.3}\\
\frac{d \sigma_{R L}^{H}\left(\alpha_{1}, \alpha_{2}, \alpha_{3}\right)}{d \cos \theta}= & \frac{\hat{t}^{2}}{8 \pi \hat{s}^{3}}\left[g_{y}^{4} y_{R \alpha_{1}}^{2} y_{L \alpha_{2}}^{2}+\frac{2}{9} g_{s}^{4}\right] \delta_{\alpha_{2} \alpha_{3}}  \tag{4.4}\\
\frac{d \sigma_{R R}^{H}\left(\alpha_{1}, \alpha_{2}, \alpha_{3}\right)}{d \cos \theta}= & \frac{\hat{u}^{2}}{8 \pi \hat{s}^{3}}\left[g_{y}^{4} y_{R \alpha_{1}}^{2} y_{R \alpha_{2}}^{2}+\frac{2}{9} g_{s}^{4}\right] \delta_{\alpha_{2} \alpha_{3}} \tag{4.5}
\end{align*}
$$

where $\hat{t}=-\hat{s} / 2(1-\cos \theta)$ and $\hat{u}=-\hat{s} / 2(1+\cos \theta)$ are the Mandelstam variables in the partonic c.m. frame. Notice the absence of $g_{s}^{2} g_{w}^{2}$ and $g_{s}^{2} g_{y}^{2}$ terms since electroweak and strong amplitudes do not interfere due to the color structure. The values of the corresponding hard overlap matrix elements in the LL channel can be obtained by equating the general form (3.5) to the values of the hard cross sections (1.2, 4.3, 4.4,4.5). Namely, from $\mathcal{O}_{L L}^{H}\left(\alpha_{1}, \alpha_{1} ; \alpha_{2}, \alpha_{2} ; \alpha_{3}, \alpha_{3}\right)=\frac{d \sigma_{L L}^{H}\left(\alpha_{1}, \alpha_{2}, \alpha_{3}\right)}{d \cos \theta}$ and defining $\alpha_{i}=\frac{g_{i}^{2}}{4 \pi}$ one obtains:

$$
\begin{array}{ll}
O_{000}^{H}=\frac{2 \pi \hat{u}^{2}}{9 \hat{s}^{3}}\left(\alpha_{s}^{2}+\frac{27}{32} \alpha_{w}^{2}+\frac{1}{288} \alpha_{y}^{2}\right) & O_{110}^{H}=\frac{\pi \hat{u}^{2}}{8 \hat{s}^{3}}\left(\alpha_{w}^{2}+\frac{1}{9} \alpha_{w} \alpha_{y}\right) \\
O_{011}^{H}=\frac{8 \pi \hat{u}^{2}}{9 \hat{s}^{3}}\left(\alpha_{s}^{2}-\frac{9}{32} \alpha_{w}^{2}+\frac{1}{288} \alpha_{y}^{2}\right) & O_{101}^{H}=-\frac{\pi \hat{u}^{2}}{2 \hat{s}^{3}}\left(\alpha_{w}^{2}-\frac{1}{9} \alpha_{w} \alpha_{y}\right) \tag{4.7}
\end{array}
$$

The same procedure allows to find the values for the RL channel (in this case we have two different overlaps: one for the initial $u_{R}$ and one for $d_{R}$, distinguished by the hypercharge contribution):

$$
\begin{equation*}
O_{00}^{H}=\frac{2 \pi \hat{t}^{2}}{9 \hat{s}^{3}}\left(\alpha_{s}^{2}+\frac{1}{8} \alpha_{y}^{2} y_{R}^{2}\right) \quad O_{11}^{H}=\frac{8 \pi \hat{t}^{2}}{9 \hat{s}^{3}}\left(\alpha_{s}^{2}+\frac{1}{8} \alpha_{y}^{2} y_{R}^{2}\right) \tag{4.8}
\end{equation*}
$$

The dressed overlap matrix can be now obtained by using the values for the hard overlap matrix of eqs. (4.6-4.8) and applying the rule of eq.(2.13). So for instance

$$
\begin{equation*}
O_{101}(s)=O_{101}^{H} \exp \left[-2 L_{W}\right]=-\frac{\pi \hat{u}^{2}}{8 \hat{s}^{3}}\left(\alpha_{w}^{2}-4 y_{L}^{2} \alpha_{w} \alpha_{y}\right) \exp \left[-2 L_{W}\right] \tag{4.9}
\end{equation*}
$$

and so on. On the other hand, the $L R, R R$ channels do not receive BN EW corrections.
The gluon gluon hard cross section $(g g \rightarrow Q \bar{Q})$ can be decomposed giving the chirality of the final states:

$$
\begin{equation*}
\frac{d \sigma^{H}}{d \cos \theta}=\frac{d \sigma_{R}^{H}}{d \cos \theta}+\frac{d \sigma_{L}^{H}}{d \cos \theta} \quad ; \quad \frac{d \sigma_{L}^{H}}{d \cos \theta}=\frac{d \sigma_{R}^{H}}{d \cos \theta} \tag{4.10}
\end{equation*}
$$

[^0]$$
1 . \times 10^{-7}
$$

Figure 3: The differential cross section for $b \bar{t}$ production, $\frac{d \sigma}{d \hat{s}}$ in $\frac{p b}{G e V^{2}}$, integrated over $\theta$ for $p_{\perp}>400 \mathrm{GeV}$. Dashed (red) line is for the fully inclusive LL BN case, while continuous (blue) line for the exclusive Sudakov case.

It is clear that the contributions coming from initial gluons, from the point of view of $S U(2)$, are two-charged-leg final states only when left handed heavy quarks are produced, while the overlap of the process $g g \rightarrow Q_{R} \bar{Q}_{R}$ is singlet under $S U(2)$ decomposition. The isospin structure of the process $g g \rightarrow Q_{L} \bar{Q}_{L}$ is given by

$$
\begin{equation*}
\frac{d \sigma_{L}^{H}\left(\alpha_{1}, \alpha_{2}\right)}{d \cos \theta}=\frac{\pi \alpha_{S}^{2}}{4 \hat{s}}\left(\frac{\hat{t}^{2}+\hat{u}^{2}}{6 \hat{t} \hat{u}}-\frac{3\left(\hat{t}^{2}+\hat{u}^{2}\right)}{8 \hat{s}^{2}}\right) \delta_{\alpha_{1} \alpha_{2}} \tag{4.11}
\end{equation*}
$$

corresponding to an overlap $\mathcal{O}^{H}$ given by the coefficients

$$
\begin{equation*}
O_{00}^{H}=\frac{1}{2} \frac{d \sigma_{L}^{H}}{d \cos \theta} \quad O_{11}^{H}=2 \frac{d \sigma_{L}^{H}}{d \cos \theta} \tag{4.12}
\end{equation*}
$$

The evolved overlap matrix and the respective dressed cross sections with the all order resummed virtual and real EW corrections, can now be obtained by using eqn.(2.10, 2.13) applied to all the hard overlap form factors. An important feature of such a channel is the fact that, being a mixture of $s$ and $t$-channels, its angular dependence is different from the $q \bar{q}$ s-channel cross section. This fact is important not only for $t \bar{t}, b \bar{b}$ production but mainly for $t \bar{b}, b \bar{t}$ production, where the tree level $\alpha_{W}^{2}$ cross section proceeds only through s-channel annihilation. The outcome is that the BN corrected angular distribution is different from the tree level one (fig. 5).

Finally, in fig. 园 we plot the differential cross section for the process $P P \rightarrow b \bar{t}+X$ for the BN and Sudakov cases.

We can obtain a rather simple formula for the BN corrections if we evaluate the hard cross section in the limit $g_{y}, g_{w} \rightarrow 0$; this is a reasonable approximation since the
contributions proportional to $g_{s}$ are the bulk of the hard cross sections. It is easy to check that in this case the same correction is obtained for the $q \bar{q}$ and $g g$ channels, allowing the factorization of the BN corrections with respect the hard QCD cross section:

$$
\begin{align*}
& \frac{d \sigma^{B N}(P P \rightarrow t \bar{t})}{d \cos \theta} \approx \frac{d \sigma^{B N}(P P \rightarrow b \bar{b})}{d \cos \theta} \approx \frac{d \sigma_{Q C D}^{H}(P P \rightarrow t \bar{t})}{d \cos \theta} \frac{1}{4}\left(3+\exp \left[-2 L_{W}(\hat{s})\right]\right)(4 .  \tag{4.13}\\
& \frac{d \sigma^{B N}(P P \rightarrow t \bar{b})}{d \cos \theta} \approx \frac{d \sigma^{B N}(P P \rightarrow b \bar{t})}{d \cos \theta} \approx \frac{d \sigma_{Q C D}^{H}(P P \rightarrow t \bar{t})}{d \cos \theta} \frac{1}{4}\left(1-\exp \left[-2 L_{W}(\hat{s})\right]\right)(4 . \tag{4.14}
\end{align*}
$$

Let us now comment on our final results for the cross sections $P P \rightarrow Q \bar{Q}+X$ where $Q \bar{Q}=t \bar{t}, t \bar{b}, b \bar{t}, b \bar{b}$, summarized in figures 团, We recall again that we consider two kinds of observables:

$$
\begin{array}{r}
\text { "Sudakov" : }(P P \rightarrow \text { tagged final state }+X) \text { with } W, Z \notin X \\
\text { " } B N^{\prime \prime}:(P P \rightarrow \text { tagged final state }+X) \text { with } W, Z \in X
\end{array}
$$

Sudakov corrections always depress the tree level cross section [8, while in BN case the sign of the corrections can be positive or negative. The results for resummed BN and Sudakov corrections to $t \bar{t}$ hadronic cross section are shown in figure $\boldsymbol{T}_{\text {. Both corrections are }}$ negative in size and more pronounced in the Sudakov case. The $b \bar{b}$ case has a very similar behavior: in fact, in the limit $\alpha_{Y} \rightarrow 0$ the $t \bar{t}$ and $b \bar{b}$ partonic cross sections are equal; the same holds in the $t \bar{b}, b \bar{t}$ case. Notice however that the $t \bar{b}$ and $b \bar{t}$ hadronic cross sections are very different, due to the different luminosities involved. For instance at tree level the hadronic cross section for $t \bar{b}$ depends on $L_{u \bar{d}}$ while the one for $b \bar{t}$ depends on $L_{d \bar{u}}$, which is smaller.

In the the $b \bar{t}, t \bar{b}$ channels the BN corrections instead enhance the cross sections. From fig. 5 we see that the BN enhancement is dramatic: the cross section including gauge bosons emission is more than one order of magnitude bigger than the exclusive (Sudakov) one. This is due to an interesting interplay between strong and weak interactions. In fact in these channels, while the tree level cross sections are proportional to $\alpha_{w}^{2}$, when considering BN corrections they receive big contribution from the strong $O\left(\alpha_{s}^{2}\right)$ channel. Moreover, the tree level and the BN cross sections have different angular behavior (see fig. 5).

For heavy quark production the leading tree level (with no $W$ emission) cross sections are of order $O\left(\alpha_{S}^{2}\right)$ when diagonal in isospin ( $\sigma_{p p \rightarrow t \bar{t}}^{H}, \sigma_{p p \rightarrow b \bar{b}}^{H}$ ), while the two isospin changing one $\sigma_{p p \rightarrow b \bar{t}}^{H}, \sigma_{p p \rightarrow t \bar{b}}^{H}$ are $O\left(\alpha_{W}^{2}\right)$. We can summarize the pattern for the leading tree level and one loop EW corrected cross sections as follows:

| Cross sections | Isospin Structure | Tree Level $(X=0)$ | BN corrections |
| :---: | :---: | :---: | :---: |
| $p p \rightarrow Q_{i} \bar{Q}_{j}+X$ | $\delta_{i j}$ | $\alpha_{s}^{2}$ | $\alpha_{s}^{2} \alpha_{w} \log ^{2} \hat{s}$ |
|  | $i \neq j$ | $\alpha_{w}^{2}$ | $\alpha_{s}^{2} \alpha_{w} \log ^{2} \hat{s}$ |

Finally, we expect analogous results for other observables in which nonabelian legs are detected, such as single top production.

In this work the CTEQ5M parton distributions [5] have been used.


Figure 4: Sudakov and BN electroweak corrections at Leading Log order for the process $P P \rightarrow t \bar{t}$ (similar results hold for $P P \rightarrow b \bar{b}$ ) in the massless limit. Top: effects of radiative corrections on $\frac{d^{2} \sigma}{d \hat{s} d \cos \theta}$ at $\theta=\frac{\pi}{2}, \hat{s}$ being the partonic c.m. energy (which is also the $t \bar{t}$ invariant mass) and $\theta$ the partonic reference frame scattering angle. Bottom: the same for $\frac{d \sigma}{d \delta}$, integrated over $\theta$ for $p_{\perp}>400$ GeV.

## 5. Conclusions

The main point of this paper is that emission of real gauge bosons should be carefully examined when considering LHC observables. As we have seen, including it or not may result in cross sections differing by an order of magnitude.

As explained in the text, a number of simplifications have been made: we consider the proton sea to be an isospin singlet, only resummed double logs are computed and so on.


Figure 5: Top: angular dependence at fixed energy ( $\sqrt{\hat{s}}=2000 \mathrm{GeV}$ in the partonic frame) of the LL BN cross section (red dashed line) and of the Sudakov one for $t \bar{b}$ production (blue continuous line). Bottom: $\sqrt{\hat{s}}$ dependence of the ratio of BN and Sudakov cross sections for $p_{\perp} \geq 400 \mathrm{GeV}$. Dashed (red) line for $b \bar{t}$, continuous (blue) line for $t \bar{b}$.

Therefore our main results in formulae (4.13:4.14) and figs 3,4 have to be taken as first order estimates. However first, more detailed calculations are feasible and not too hard and second, we think that the outcome is already clear at this preliminary level: by considering observables that are inclusive, rather than exclusive, of weak bosons emissions, the pattern of radiative electroweak corrections changes significantly. In some cases the cross sections that one wants to measure are drammaticaly enhanced (fig.4), and also differential cross sections such as the angular distribution are significantly different (fig. 4).

While we reckon that it is not entirely clear what can, and will, be measured at the LHC with respect to "soft" final Ws and Zs emission, we think that it is worthwhile opening the physics case.

## Acknowledgments

We thank M. Ciafaloni for useful theoretical discussions and M. Grazzini for help on QCD Structure Functions. D.C. acknowledges Cern Theory Division where he started this work.

## References

[1] P. Ciafaloni and D. Comelli, Sudakov enhancement of electroweak corrections, Phys. Lett. B 446 (1999) 278 hep-ph/9809321.
[2] S. Dittmaier and M. Kramer, Electroweak radiative corrections to w-boson production at hadron colliders, Phys. Rev. D 65 (2002) 073007 hep-ph/0109062;
U. Baur, O. Brein, W. Hollik, C. Schappacher and D. Wackeroth, Electroweak radiative corrections to neutral-current drell- yan processes at hadron colliders, Phys. Rev. D 65
(2002) 033007 hep-ph/0108274;
E. Maina, S. Moretti, M.R. Nolten and D.A. Ross, Phys. Lett. B 570 (2003) 205;
M. Beccaria, F.M. Renard and C. Verzegnassi, Special features of heavy quark-antiquark pair production ratios at LHC, hep-ph/0405036;
U. Baur and D. Wackeroth, Electroweak radiative corrections to p $\bar{p} \rightarrow W^{+-} \rightarrow L^{+-} \nu$ beyond the pole approximation, Phys. Rev. D 70 (2004) 073015 hep-ph/0405191;
J.H. Kuhn, A. Kulesza, S. Pozzorini and M. Schulze, Logarithmic electroweak corrections to hadronic $z+1$ jet production at large transverse momentum, Phys. Lett. B 609 (2005) 277 hep-ph/0408308;
E. Accomando, A. Denner and A. Kaiser, Logarithmic electroweak corrections to gauge-boson pair production at the LHC, Nucl. Phys. B 706 (2005) 325 .
E. Accomando and A. Kaiser, Electroweak corrections and anomalous triple gauge-boson couplings in $W W$ and $W$ Z production at the LHC, Phys. Rev. D 73 (2006) 093006 hep-ph/0511088;
E. Accomando, A. Denner and A. Kaiser, Logarithmic electroweak corrections to gauge-boson pair production at the LHC, Nucl. Phys. B 706 (2005) 325 (hep-ph/0409247];
W. Hollik et al., Electroweak physics, Acta Phys. Polon. B 35 (2004) 2533;
S. Moretti, M.R. Nolten and D.A. Ross, Weak corrections and high $E_{t}$ jets at Tevatron, hep-ph/0503152; Weak effects in proton beam asymmetries at polarised RHIC and beyond, hep-ph/0509254; Weak corrections to gluon-induced top-antitop hadro- production, Phys. Lett. B 639 (2006) 513 hep-ph/0603083;
M. Beccaria, P. Ciafaloni, D. Comelli, F. M. Renard, C. Verzegnassi, Logarithmic expansion of electroweak corrections to four-fermion processes in the TeV region, Phys. Rev. D 61
(2000) 073005; The role of the top mass in b production at future lepton colliders, Phys. Rev. D 61 (2000) 011301 ;
M. Beccaria, F. M. Renard, C. Verzegnassi, Logarithmic SUSY electroweak effects on four-fermion processes at TeV energies, Phys. Rev. D 63 (2001) 095010 Top quark production at future lepton colliders in the asymptotic regime, Phys. Rev. D 63 (2001)
053013; The role of universal and non universal sudakov logarithms in four fermion processes at TeV energies: the one-loop approximation revisited, Phys. Rev. D 64 (2001) 073008
hep-ph/0103335; Reliability of a high energy one-loop expansion of $e^{+} e^{-} \rightarrow W+W-$ in the SM and in the MSSM, Nucl. Phys. B 663 (2003) 394 hep-ph/0304175;
M. Beccaria, S. Prelovsek, F.M. Renard and C. Verzegnassi, Top quark production at TeV energies as a potential SUSY detector, Phys. Rev. D 64 (2001) 053016 hep-ph/0104245; M. Beccaria, M. Melles, F. M. Renard, C. Verzegnassi, SUSY scalar production in the electroweak Sudakov regime of lepton colliders, Phys. Rev. D 65 (2002) 093007 . M. Beccaria, M. Melles, F.M. Renard, S. Trimarchi and C. Verzegnassi, Sudakov expansions at one loop and beyond for charged scalar and fermion pair production in SUSY models at future linear colliders, Int. J. Mod. Phys. A 18 (2003) 5069 hep-ph/0304110;
M. Beccaria, F. M. Renard, S. Trimarchi, C. Verzegnassi, Charged Higgs production in the $1-\mathrm{TeV}$ domain as a probe of supersymmetric models, Phys. Rev. D 68 (2003) 035014 , M. Beccaria, F.M. Renard and C. Verzegnassi, Reliability of a high energy one-loop expansion of $e^{+} e^{-} \rightarrow W+W-$ in the SM and in the MSSM, Nucl. Phys. B 663 (2003) 394 hep-ph/0304175;
A. Denner and S. Pozzorini, One-loop leading logarithms in electroweak radiative corrections. i: results, Eur. Phys. J. C 18 (2001) 461 hep-ph/0010201;
S. Pozzorini, Electroweak radiative corrections at high energies, hep-ph/0201077,
H. Hori, H. Kawamura, J. Kodaira, Electroweak Sudakov at two loop level, Phys. Lett. B 491 (2000) 275;
W. Beenakker and A. Werthenbach, New insights into the perturbative structure of electroweak sudakov logarithms: breakdown of conventional exponentiation, Phys. Lett. B 489 (2000) 148 hep-ph/0005316;
A. Denner and S. Pozzorini, One-loop leading logarithms in electroweak radiative corrections, I: Results, Eur. Phys. J. C 18 (2001) 461 and Eur. Phys. J. C 21 (2001) 63;
W. Beenakker and A. Werthenbach, Electroweak two-loop Sudakov logarithms for on-shell fermions and bosons, Nucl. Phys. B 630 (2002) 3;
A. Denner, M. Melles and S. Pozzorini, Two-loop electroweak angular-dependent logarithms at high energies, Nucl. Phys. B 662 (2003) 299 hep-ph/0301241;
U. Aglietti and R. Bonciani, Master integrals with one massive propagator for the two- loop electroweak form factor, Nucl. Phys. B 668 (2003) 3 hep-ph/0304028;
B. Feucht, J.H. Kuhn and S. Moch, Fermionic and scalar corrections for the abelian form factor at two loops, Phys. Lett. B 561 (2003) 111 hep-ph/0303016;
B. Feucht, J. H. Kuhn, A. A. Penin and V. A. Smirnov, Two-loop Sudakov form factor in a theory with mass gap, Phys. Rev. Lett. 93 (2004) 101802;
A. Denner and S. Pozzorini, An algorithm for the high-energy expansion of multi-loop diagrams to next-to-leading logarithmic accuracy, Nucl. Phys. B 717 (2005) 48 hep-ph/0408068;
S. Pozzorini, Next-to-leading mass singularities in two-loop electroweak singlet form factors, Nucl. Phys. B 692 (2004) 135 hep-ph/0401087;
B. Jantzen, J.H. Kuhn, A.A. Penin and V.A. Smirnov, Two-loop electroweak logarithms, Phys. Rev. D 72 (2005) 051301 hep-ph/0504111;
J.H. Kuhn, A. Kulesza, S. Pozzorini and M. Schulze, Electroweak corrections to hadronic photon production at large transverse momenta, JHEP 03 (2006) 059 hep-ph/0508253; B. Jantzen, J.H. Kuhn, A.A. Penin and V.A. Smirnov, Two-loop electroweak logarithms in four-fermion processes at high energy, Nucl. Phys. B 731 (2005) 188 hep-ph/0509157; B. Jantzen and V.A. Smirnov, The two-loop vector form factor in the sudakov limit, Eur. Phys. J. C 47 (2006) 671 hep-ph/0603133.
[3] V. S. Fadin, L. N. Lipatov, A. D. Martin and M. Melles, Resummation of double logarithms
in electroweak high energy processes, Phys. Rev. D 61 (2000) 094002 ;
P. Ciafaloni, D. Comelli, Electroweak Sudakov form factors and nonfactorizable soft QED effects at NLC energies, Phys. Lett. B 476 (2000) 49;
M. Melles, Subleading sudakov logarithms in electroweak high energy processes to all orders, Phys. Rev. D 63 (2001) 034003 [hep-ph/0004056]; Resummation of Yukawa enhanced and subleading sudakov logarithms in longitudinal gauge boson and Higgs production, Phys. Rev. D 64 (2001) 014011 hep-ph/0012157; Electroweak renormalization group corrections in high energy processes, Phys. Rev. D 64 (2001) 054003 hep-ph/0102097; Resummation of angular dependent corrections in spontaneously broken gauge theories, Eur. Phys. J. C 24 (2002) 193 hep-ph/0108221]; Electroweak radiative corrections in high energy processes, Phys. Rept. 375 (2003) 219 hep-ph/0104232];
J.H. Kuhn, A.A. Penin and V.A. Smirnov, Summing up subleading sudakov logarithms, Eur. Phys. J. C 17 (2000) 97 hep-ph/9912503;
J.H. Kuhn, S. Moch, A.A. Penin and V.A. Smirnov, Next-to-next-to-leading logarithms in four-fermion electroweak processes at high energy, Nucl. Phys. B 616 (2001) 286 hep-ph/0106298.
[4] M. Ciafaloni, P. Ciafaloni and D. Comelli, Bloch-nordsieck violating electroweak corrections to inclusive TeV scale hard processes, Phys. Rev. Lett. 84 (2000) 4810 hep-ph/0001142;
Electroweak double logarithms in inclusive observables for a generic initial state, Phys. Lett.
B 501 (2001) 216 hep-ph/0007096;
Nucl. Phys. B 589 (2000) 359;
Enhanced electroweak corrections to inclusive boson fusion processes at the TeV scale, Nucl. Phys. B 613 (2001) 382;
P. Ciafaloni, D. Comelli and A. Vergine, Sudakov electroweak effects in transversely polarized beams, JHEP 07 (2004) 039 hep-ph/0311260.
[5] M. Ciafaloni, P. Ciafaloni and D. Comelli, Phys. Rev. Lett. 87 (2001) 211802;
M. Ciafaloni, P. Ciafaloni, D. Comelli, Nucl. Phys. B 589 (2000) 359.
[6] M. Ciafaloni, P. Ciafaloni and D. Comelli, Towards collinear evolution equations in electroweak theory, Phys. Rev. Lett. 88 (2002) 102001 hep-ph/0111109;
P. Ciafaloni and D. Comelli, Electroweak evolution equations, JHEP 11 (2005) 022 hep-ph/0505047.
[7] QCD and collider physics, R.K. Ellis, W.J. Stirling, B.R. Webber, T. Ericson, P.Y. Landshoff eds, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology.
[8] M. Beccaria, S. Bentvelsen, M. Cobal, F.M. Renard and C. Verzegnassi, Special supersymmetric features of large invariant mass unpolarized and polarized top antitop production at LHC, Phys. Rev. D 71 (2005) 073003 hep-ph/0412249;
S. Moretti, M.R. Nolten and D.A. Ross, Weak corrections to gluon-induced top-antitop hadroproduction, Phys. Lett. B 639 (2006) 513 hep-ph/0603083.
[9] CTEQ collaboration, H.L. Lai et al., Global QCD analysis of parton structure of the nucleon: CTEQ5 parton distributions, Eur. Phys. J. C 12 (2000) 375 hep-ph/9903282.


[^0]:    ${ }^{*}$ We do not consider here t-channel contributions from initial sea heavy quarks like $b \bar{t} \rightarrow b \bar{t}$, a process initiated by the excitations of bottom and top quarks from the gluon sea. However for a full consistent calculation a careful evaluation of these kind of processes has to be included.

